Exam 2 Solutions

Tuesday, July 13, 2021 1:21 PM

Problem 1:

$f(\theta) = -\frac{1}{2}\sin(3\theta - \pi).$

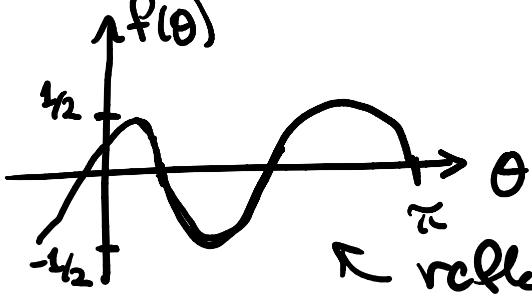
Amplitude $= \frac{1}{2} \rightarrow$ compresses the graph by half

Period $= \frac{2\pi}{3} \rightarrow$ graph is compressed horizontally by a third

Vertical shift $= 0$

Horizontal shift $= \frac{\pi}{3}$ to the right.

entire graph is shifted to the right.



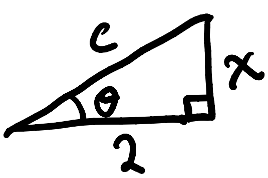
reflected about x axis

Problem 2.

$\cos(\arctan(\frac{x}{2}))$

Let $\theta = \arctan(\frac{x}{2})$

$\tan\theta = \frac{x}{2}$



Pythag. Theorem: $c^2 = x^2 + 2^2$

$c = \sqrt{x^2+4}$

Then $\boxed{\cos\theta = \frac{2}{\sqrt{x^2+4}}}$

$$\boxed{\cos\theta = \frac{\quad}{\sqrt{x^2+4}}}$$

Problem 3.

$$\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$$

Use the sum identity for cosine:

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B),$$

then

$$\cos\left(\frac{\pi}{2}+\theta\right) = \underbrace{\cos\left(\frac{\pi}{2}\right)}_{=0}\cos(\theta) - \underbrace{\sin\left(\frac{\pi}{2}\right)}_{=1}\sin(\theta)$$

from unit circle

$$= -1\cdot\sin\theta$$

$$\cos(\pi/2+\theta) = -\sin\theta \quad ✓$$

The identity has been verified.

This means that sine & cosine graphs are related in that horizontally shifting cosine is equivalent to reflecting sine about the x-axis.