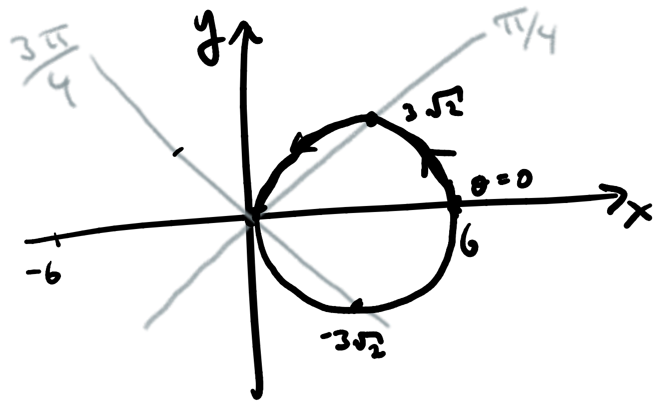
Trig S21 Practice Final Solutions

Tuesday, May 11, 2021 12:08 PM

Problem 1:

$$r = 6\cos\theta.$$



θ	r
0	6
$\frac{\pi}{4}$	$6\cdot\frac{\sqrt{2}}{2} = 3\sqrt{2}$
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	$6\cdot\left(-\frac{\sqrt{2}}{2}\right) = -3\sqrt{2}$
π	

Convert to rectangular: first multiply both sides by r.

$$\underbrace{r^2}_{=x^2+y^2} = 6\underbrace{r\cos\theta}_{=x}$$

$$\boxed{x^2+y^2 = 6x.}$$

Problem 2:

Double angle identity

$$\cos(2x) - 3\cos(x) = -2.$$

$$2\cos^2(x) - 1 - 3\cos(x) = -2$$

$$+2 \qquad +2$$

$$2\cos^2(x) - 3\cos(x) + 1 = 0.$$

Quadratic formula:

$$\cos(x) = \frac{+3 \pm \sqrt{9-4(2)(1)}}{4}$$

$$\cos(x) = \frac{\quad}{4}$$

$$= \frac{3 \pm \sqrt{9-8}}{4}$$

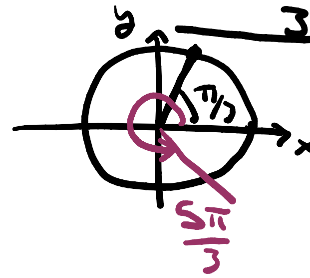
$$\cos(x) = \frac{3 \pm 1}{4}$$

$$\cos(x) = \frac{4}{4} \quad \text{or} \quad \cos(x) = \frac{2}{4}$$

$$\cos(x) = 1 \qquad\qquad \cos(x) = \frac{1}{2}$$

$$\boxed{x = 0 + 2\pi k}$$

$$\text{or} \quad \boxed{\begin{array}{l} x = \frac{\pi}{3} + 2\pi k \\ x = \frac{5\pi}{3} + 2\pi k. \end{array}}$$



Problem 3:

$$z = 2(\cos(30^\circ) + i\sin(30^\circ)).$$

Use unit circle: $\cos(30^\circ) = \frac{\sqrt{3}}{2}$

$\sin(30^\circ) = \frac{1}{2}$

$$z = 2\left(\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2}\right)$$

$$\boxed{z = \sqrt{3} + i.}$$

Graph: Im

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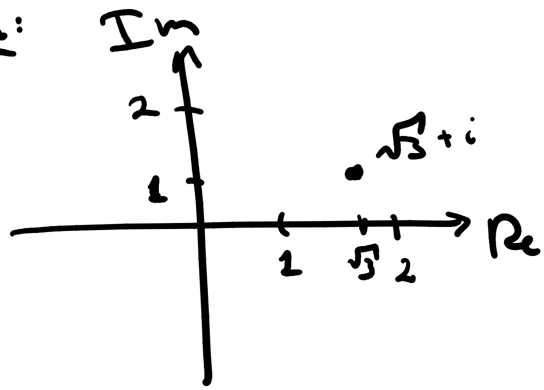
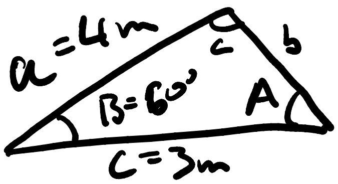


Problem 4:



use law of cosines & find b:

$$b^2 = a^2 + c^2 - 2ac\cos B)$$

$$b^2 = 4^2 + 3^2 - 2\cdot 4\cdot 3\cos(0^\circ).$$

$$= 16 + 9 - 24 \cdot \frac{1}{2}$$

$$b^2 = 25 - 12$$

$$\boxed{b = \sqrt{13}}$$

Then use law of sines to find A:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{4} = \frac{\sin(60^\circ)}{\sqrt{13}}$$

$$\sin A = \cancel{4} \cdot \frac{\sqrt{3}}{2\sqrt{13}}$$

$$\sin A = 2\sqrt{\frac{3}{13}}$$

$$\boxed{A = \sin^{-1}\left(2\sqrt{\frac{3}{13}}\right)}$$

$$\boxed{A = \sin^{-1}\left(2\sqrt{13}\right)}$$

Last find C with

$$A + B + C = 180°$$

$$C = 180° - B - A$$

$$= 180° - 60° - \sin^{-1}\left(2\sqrt{\frac{3}{13}}\right)$$

$$\boxed{C = 60° - \sin^{-1}\left(2\sqrt{\frac{3}{23}}\right)}$$

Problem 5.

$$\vec{u} = \langle 1, 0 \rangle, \quad \vec{v} = \langle 2, 3 \rangle.$$

$$3\vec{u} - \vec{v} = 3\langle 1, 0 \rangle - \langle 2, 3 \rangle$$

$$= \langle 3, 0 \rangle - \langle 2, 3 \rangle$$

$$= \langle 1, -3 \rangle.$$

To find the unit vector, first find the magnitude:

$$|3\vec{u} - \vec{v}| = \sqrt{1^2 + (-3)^2}$$

$$= \sqrt{1 + 9}$$

$$= \sqrt{10}.$$

Unit vector is

$$\frac{3\vec{u} - \vec{v}}{|3\vec{u} - \vec{v}|} = \frac{\langle 1, -3 \rangle}{\sqrt{10}}$$

$$\langle 1 \quad -3 \rangle$$

$|3\vec{u} - \vec{v}| \ldots$

$$\boxed{= \left\langle \frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}} \right\rangle}$$

Angle between $\vec{u}$ & $3\vec{u}-\vec{v}$:

$$\vec{u} \cdot (3\vec{u}-\vec{v}) = |\vec{u}|\,|3\vec{u}-\vec{v}|\cos\theta$$

First find dot product:

$$\vec{u} \cdot (3\vec{u}-\vec{v}) = \langle 1, 0\rangle \cdot \langle 1, -3\rangle$$
$$= 1 + 0$$
$$= 1.$$

$$|\vec{u}| = \sqrt{1^2 + 0^2}$$
$$= 1$$

and $|3\vec{u}-\vec{v}| = \sqrt{10}$ as just found.

Plugging these in:

$$1 = 1 \cdot \sqrt{10}\cos\theta$$

$$\cos\theta = \frac{1}{\sqrt{10}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right)}$$