Trig S21 Practice Exam 2

Wednesday, March 17, 2021 8:25 PM

Problem 1.

$$f(\theta) = \frac{1}{2}\sin\left(\frac{\pi}{2}(\theta+1)\right) - 1.$$

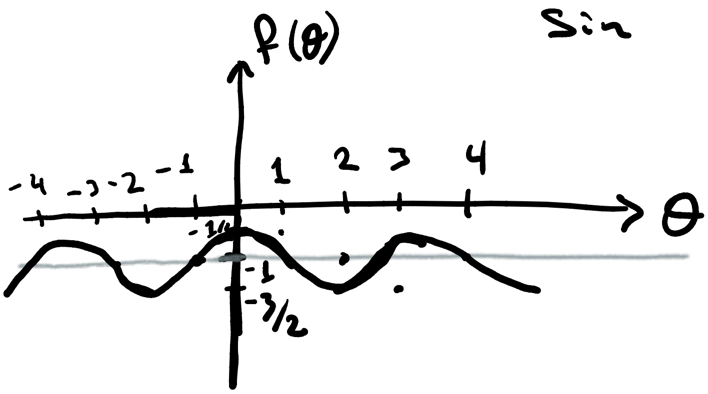
The vertical shift is 1 down.

the amplitude is 1/2

The period is $\dfrac{2\pi}{\frac{\pi}{2}} = 2\pi \cdot \dfrac{2}{\pi} = 4.$

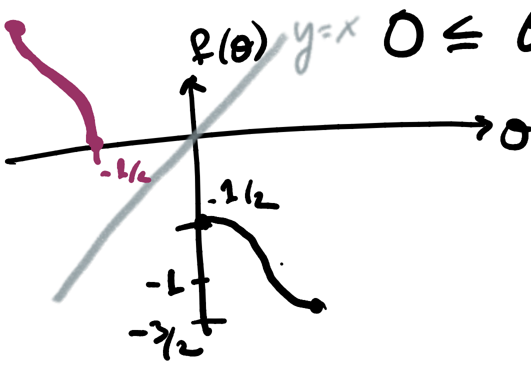
The horizontal shift is 1 to the left.

There are no asymptotes, since sin has none.



From the graph, we see there are no zeros.

To graph the inverse function, we restrict this function to $0 \le \theta \le 2$, (other choices are ok too!)



Flipping around the line $y=x$ gives us the red graph on top left.

on top left.

There is a systematic way to find the inverse function, but notice that the usual $\sin^{-1}$ graph has been shifted left and up, and stretched a bit, we might guess something like

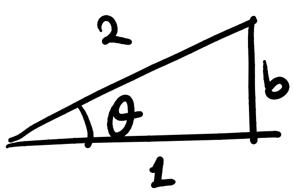
$$f^{-1}(y) = 1 + \frac{2}{\pi}\sin^{-1}(2y+1).$$

Problem 2.

$$\sin(\cos^{-1}(1/2)).$$

Let $\theta = \cos^{-1}(1/2)$.

Then $\cos\theta = 1/2$.



using SOHCAHTOA, $\frac{adj}{hyp} = \frac{1}{2}$

To find b,

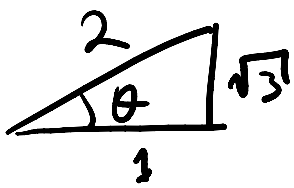
use Pythag:

$$1^2 + b^2 = 2^2$$

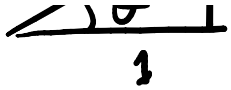
$$1 + b^2 = 4$$

$$b^2 = 4 - 1$$

$$b = \sqrt{3}.$$



Then $\sin\theta = \frac{\sqrt{3}}{2}$,

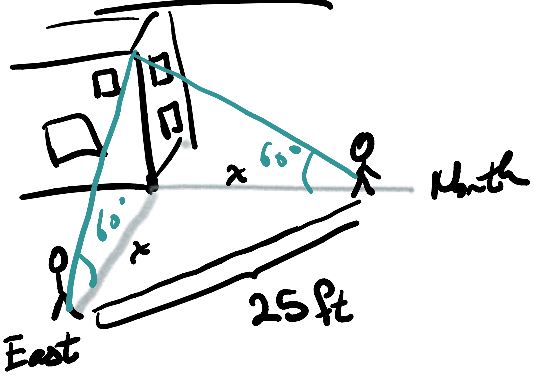


Then $\sin\theta = \frac{\sqrt{3}}{2}$,

which is the same as

$$\boxed{\sin\left(\cos^{-1}\left(\tfrac{1}{2}\right)\right) = \sqrt{3}/2.}$$

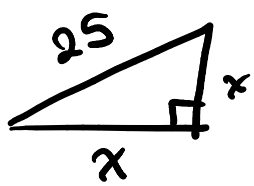
Problem 3.



★ We need one more piece of information to solve this problem: that you are both the same distance from the building.

Call that distance x.

The triangle on the ground is:



We can find x with Pythag:

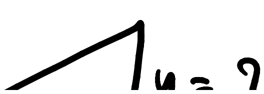
$$x^2 + x^2 = 25$$

$$2x^2 = 25$$

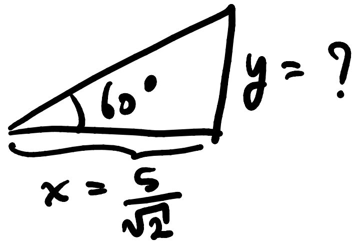
$$x^2 = \frac{25}{2}$$

$$x = \sqrt{\frac{25}{2}} = \frac{5}{\sqrt{2}} \text{ ft.}$$

Then, the triangle from you to the top of the building is



Using SOHCAHTOA



Using SOHCAHTOA,

$$\frac{5}{\sqrt{2}} \tan(60^\circ) = \frac{y}{\frac{5}{\sqrt{2}}} \cdot \frac{5}{\sqrt{2}}$$

$$y = \frac{5}{\sqrt{2}} \tan(60^\circ).$$

From the unit circle,

$$\tan(60^\circ) = \frac{\sin(60^\circ)}{\cos(60^\circ)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}.$$

So $\boxed{y = \frac{5}{\sqrt{2}}\sqrt{3} \text{ ft.}}$

Problem 4. Mistake in this problem originally.

$$\frac{1}{\csc\theta}\left(\cot\theta - \cos\theta\cot\theta - \sin\theta\right)$$

$$= \sin\theta\left(\frac{\cos\theta}{\sin\theta} - \underbrace{\cos\theta \cdot \frac{\cos\theta}{\sin\theta} - \sin\theta}_{\text{add these terms as fractions:}}\right)$$

$$-\frac{\cos^2\theta}{\sin\theta} - \sin\theta \cdot \frac{\sin\theta}{\sin\theta}$$

$$= \frac{-\cos^2\theta - \sin^2\theta}{\sin\theta}$$

$$= \frac{-\cos^2\theta - \sin^2\theta}{\sin\theta}$$

$$= \frac{-1}{\sin\theta}$$

$$= \cancel{\sin\theta}\left(\frac{\cos\theta}{\cancel{\sin\theta}} - \frac{1}{\cancel{\sin\theta}}\right)$$

$$= \cos\theta - 1.$$

This function has amplitude 1, period 2π, & vertical shift 1 down.